Summation Notation Practice

(a) Fill in the blanks so the sums are equal

$$\sum_{k=1}^{5} 2^k = \sum_{k=\_}^{7} 2^{\_} = 2\sum_{k=\_}^{\_} 2^k$$

(b) Simplify: $\sum_{n=1}^{100}(n^3-n^2) - \sum_{n=45}^{100}(n^3-n^2-n) - \sum_{n=1}^{100} n$

(c) Write in sigma notation:

$$\frac{1}{5} - \frac{1}{10} + \frac{1}{15} - \frac{1}{20} + \frac{1}{25}$$

6/8/25

(a) $2+2^2+2^3+2^4+2^5 = 2^{3-2}+2^{4-2}+\dots+2^{7-2}$

$= 2(1+1\cdot 2^1+1\cdot 2^2+1\cdot 2^3+1\cdot 2^4)$

$\therefore \sum_{k=1}^{5} 2^k = \sum_{k=3}^{7} 2^{k-2} = 2\sum_{k=0}^{4} 2^k$

(b) $\sum_{n=1}^{100}(n^3-n^2) - \sum_{n=45}^{100}(n^3-n^2-n) - \sum_{n=1}^{100} n$

$= \sum_{n=1}^{100} n^3 - \sum_{n=1}^{100} n^2 - \sum_{n=45}^{100} n^3 + \sum_{n=45}^{100} n^2 + \sum_{n=45}^{100} n - \sum_{n=1}^{100} n$

$= \sum_{n=1}^{44} n^3 - \sum_{n=1}^{44} n^2 - \sum_{n=1}^{44} n$

$= \sum_{n=1}^{44}(n^3-n^2-n)$

$\frac{1}{5}-\frac{1}{10}+\frac{1}{15}-\frac{1}{20}+\frac{1}{25}$

$= \sum_{k=1}^{5} \frac{1}{5}\cdot\frac{(-1)^{k+1}}{k}$

$= \sum_{k=1}^{5} (-1)^{k+1}\frac{1}{5k}$